

SIMPLE 3-AXIS FLUXGATE MAGNETOMETER SYSTEM
Datasheet and user manual V4



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1. Features and technical data

Features:

- 0.1 nT sensitivity per each axis with up to 33 Hz sample rate;
- IP68 protected (up to 5 meter* under water **only FG-33**);
- GPS and automatic SD card data logger with USB mass storage interface inside**;
- Compact, light, precise and easy to use.

Applications: precision flight/marine magnetometer/compass, archeology/geology magnetometer with GPS and automatic SD card data logger;

Technical data

Parameter	MIN	TYP	MAX	Units
Power supply requirements				
Supply voltage range	7	12	16	V
Supply current (from 8V to 12V supply)				
– basic design	42	51	56	mA
– extended design	108	125	220	
Measurements				
Magnetic field sensor sensitivity		0.1		nT
Magnetic field range per each axis	+/-80000			nT
Temperature sensor sensitivity		0.1		°C
Temperature measurement range	-40		+125	°C
USART data output rate (3-axis + temperature)				
– serial capture mode	2	3	4	Hz
– simultaneous capture mode	28	33	39	
Data to uSD card rate (3-axis + temperature) ***				
– serial capture mode	2	3	4	Hz
– simultaneous capture mode	28	33	39	
uSD card capacity ***	2	4**	8	GB
Mechanical & Environment				
Length		100		mm
Width		100		
Height		90		
Weight		320		g
Water resistance			20	m
Environment temperature range	-20		+70	°C

* On request also more meters

** For extended design only (not supported by basic design)

*** Capacity of uSD card preinstalled into extended design by default

2. Overview

A simple 3-axis fluxgate magnetometer system consists of 3x orthogonal FG-3+ fluxgate sensors connected to processor board based on powerful STM32F401 MCU and NCP18WB333 temperature sensor, both mounted into waterproof plastic enclosure.

A simple 3-axis fluxgate magnetometer system usually produced in two designs: FG-33 basic (fig.2-1) and FG-33+ extended (fig.2-2). Basic design magnetometer has USART interface only to communicate with external devices, e.g. MCU, PC, Arduino, Raspberry boards, etc.



Fig.2-1. Basic design (USART only)

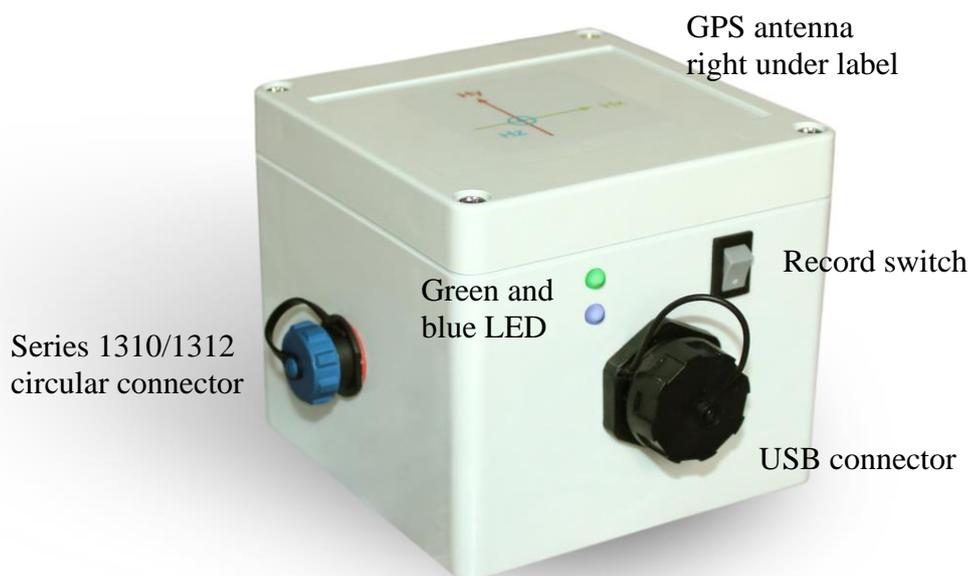


Fig.2-2. Extended design (USART+GPS+SDIO+USB MSC)

The top side of the enclosure has a magnetic field sensor axis diagram, indicating the orientation of FG-3+ sensors inside the enclosure. Z-axis sensor is oriented vertically, from bottom to top of the enclosure

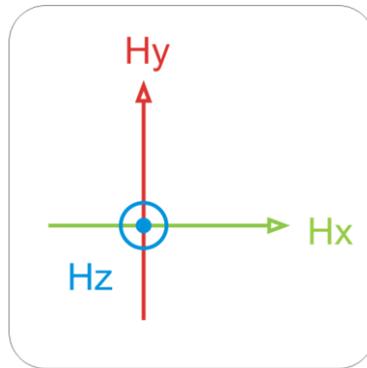


Fig.2-3. Magnetic field sensor axis diagram on the top side of enclosure

In addition to USART interface, the extended design magnetometer has the following built-in features:

- GPS module uBlox NEO-6M internally connected to STM32F401 MCU by second USART interface. GPS antenna placed right under magnetic field sensor axis diagram on the top side of enclosure;
- microSD card holder with 4GB card preinstalled, connected to STM32F401 MCU by SDIO interface;
- USB Mass Storage Class interface in order to connect to PC as convenient 4GB-capacity flash disk without another special software or driver requirement;
- a green (PWR) LED indicating if fault of the power state occurred to prevent any damage of uSD card data and blue (GPS) LED indicating the GPS status (see detailed description in section 5).

3. Pinout and connection diagrams.

Warning! There are 2. Type of connector

3.a version male connector

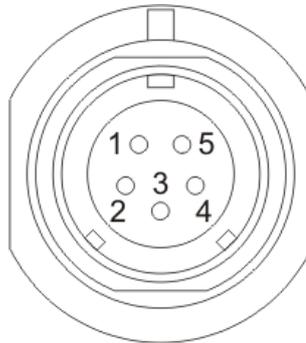


Fig.3-1a. Series 1310/1312 male circular connector pinout diagram

1 – USART TXD, 2 – USART RXD, 3 – 12V INPUT, 4 – GND, 5 – 3V3 TTL OUTPUT

3.b version female connector

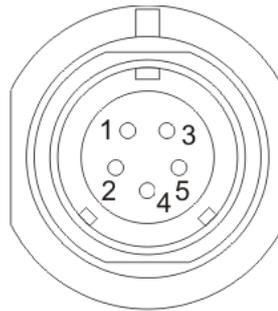


Fig.3-1b. Series 1310/1312 female circular connector pinout diagram

1 – USART TXD, 2 – USART RXD, 3 - 3V3 TTL OUTPUT, 4 – 12V INPUT, 5 – GND

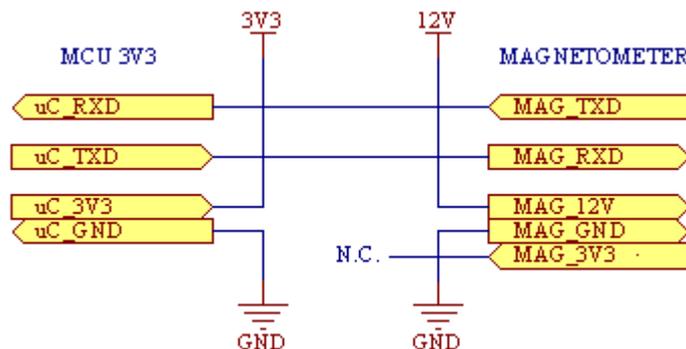


Fig.3-2. Example diagram of MCU with 3V3 TTL USART connection

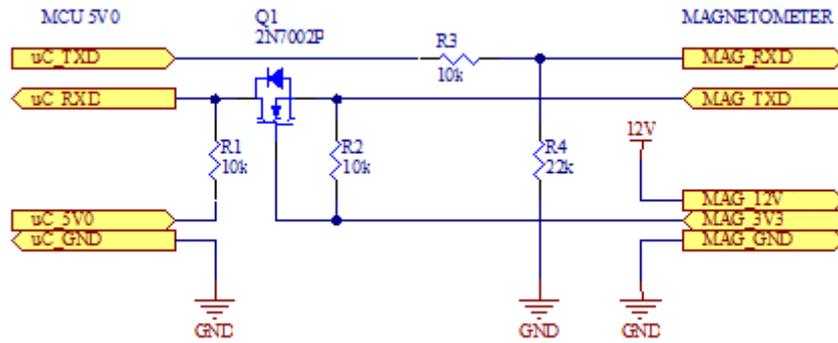


Fig.3-3. Example diagram of MCU with 5V TTL USART connection

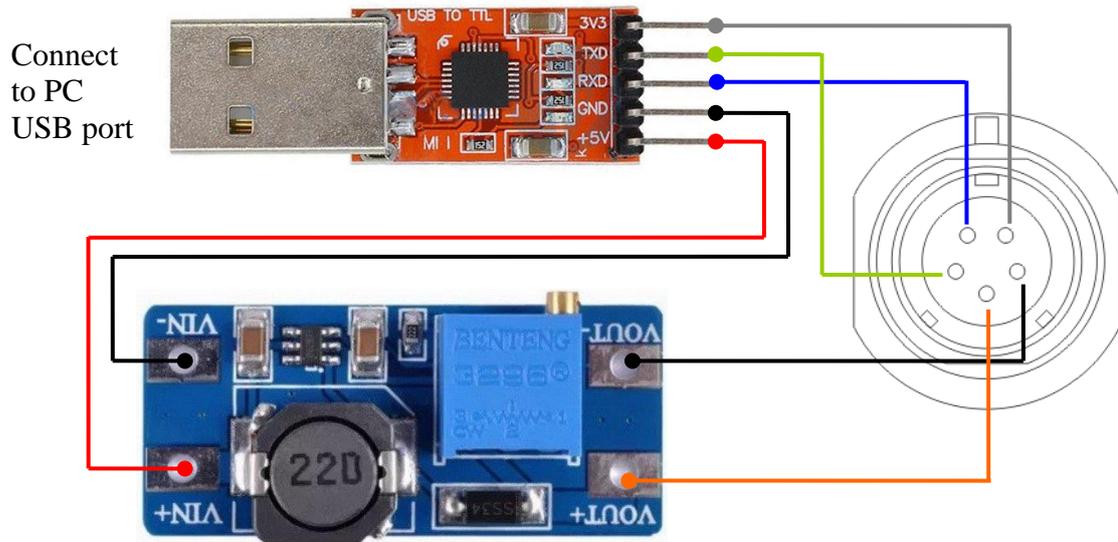
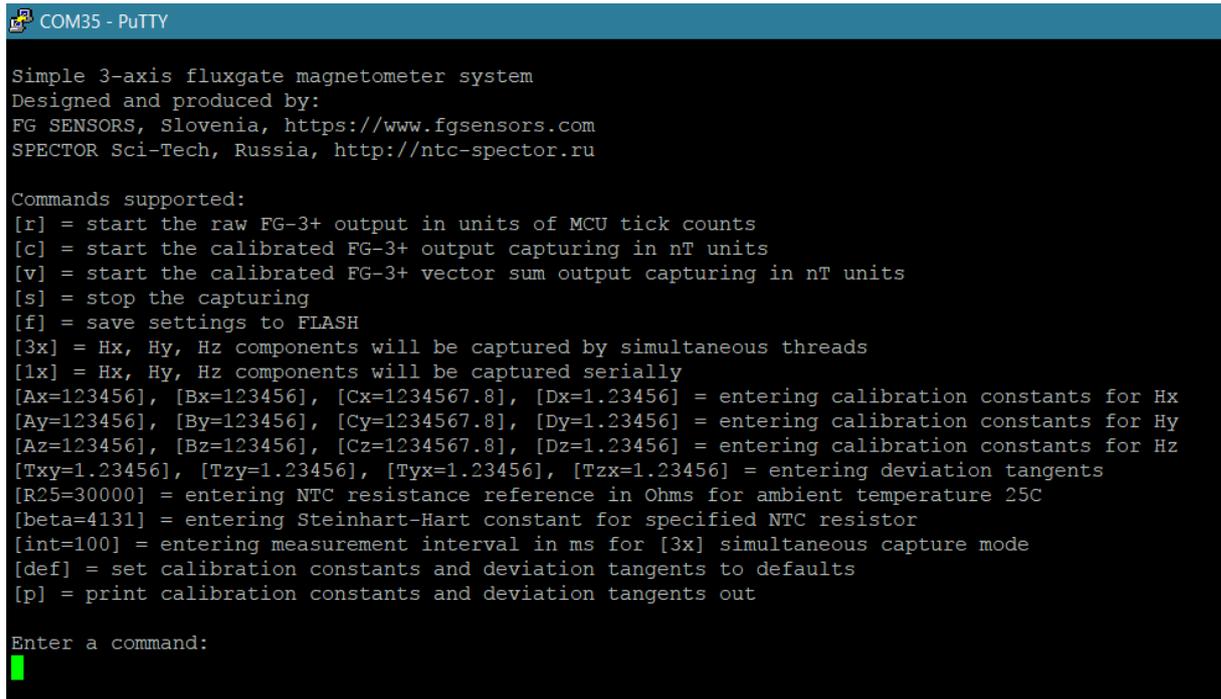


Fig.3-4. Example diagram of PC connection
using CP2102 USB-UART converter and MT3608 boost converter

Connecting to PC according to fig.3-4 follow carefully the sequence of steps below

- 3.1. Before connecting to magnetometer set the MT3608 output voltage between 7-16V;
- 3.2. Connect MT3608 to magnetometer as shown in the fig.3-4 by black and orange wires;
- 3.3. Connect CP1202 to MT3608 as shown in the fig.3-4 by black and red wires;
- 3.4. Connect CP2102 to magnetometer as shown in the fig.3-4 by blue, green and gray wires;
- 3.5. Connect CP1202 to PC, open specified COM port in terminal with baud rate 115200 bps;
- 3.6. Put any character into terminal window.

4. USART command system



```
COM35 - PuTTY
Simple 3-axis fluxgate magnetometer system
Designed and produced by:
FG SENSORS, Slovenia, https://www.fgsensors.com
SPECTOR Sci-Tech, Russia, http://ntc-spector.ru

Commands supported:
[r] = start the raw FG-3+ output in units of MCU tick counts
[c] = start the calibrated FG-3+ output capturing in nT units
[v] = start the calibrated FG-3+ vector sum output capturing in nT units
[s] = stop the capturing
[f] = save settings to FLASH
[3x] = Hx, Hy, Hz components will be captured by simultaneous threads
[1x] = Hx, Hy, Hz components will be captured serially
[Ax=123456], [Bx=123456], [Cx=1234567.8], [Dx=1.23456] = entering calibration constants for Hx
[Ay=123456], [By=123456], [Cy=1234567.8], [Dy=1.23456] = entering calibration constants for Hy
[Az=123456], [Bz=123456], [Cz=1234567.8], [Dz=1.23456] = entering calibration constants for Hz
[Txy=1.23456], [Tzy=1.23456], [Tyx=1.23456], [Tzx=1.23456] = entering deviation tangents
[R25=30000] = entering NTC resistance reference in Ohms for ambient temperature 25C
[beta=4131] = entering Steinhart-Hart constant for specified NTC resistor
[int=100] = entering measurement interval in ms for [3x] simultaneous capture mode
[def] = set calibration constants and deviation tangents to defaults
[p] = print calibration constants and deviation tangents out

Enter a command:
█
```

Fig.4-1. Simple 3-axis fluxgate magnetometer command reference in terminal window

Command is a char or sequence of chars sent to simple 3-axis fluxgate magnetometer system at once via USART. Commands can be received from PC if connections provided as shown in the fig.3-4, or from user MCU if connections provided as shown in fig.3-2 and fig.3-3.

The following communication settings are required: 115200 bps, 8 bit word length, 1 stop bit, no parity and no flow control required.

The following commands are supported.

4.1. Command «r» starts the raw magnetic field measurements output in units of built-in MCU tick counts (see «The input capture» in p.27 of AN4667 rev.3 «General-purpose timer cookbook for STM32 microcontrollers»).

Answer generated by following instruction each time when capture completed:

```
sprintf(UART1txString, "Tx=%d; Ty=%d; Tz=%d; t=%d;\n\r", Tx, Ty, Tz, t_adc_code);
```

where T_x is period captured from x-sensor, T_y is period captured from y-sensor, T_z is period captured from z-sensor, t_adc_code is adc code of voltage measured on NCP18WB333 temperature sensor.

4.2. Command «c» starts the calibrated magnetic field measurements output with 0.1 nT resolution. Answer generated by following instruction each time when capture completed:

```
sprintf(UART1txString, "Hx=%f; Hy=%f; Hz=%f; t=%f;\n\r", Hx, Hy, Hz, temperature);
```

where H_x , H_y , H_z , *temperature* are magnetic field components and temperature measured respectively.

4.3. Command «v» starts the vector sum output with 0.1 nT resolution. Answer generated by following instruction each time when capture completed:

```
sprintf(UART1txString, "H=%f; t=%f;\n\r", H, temperature);
```

where $H = Hypot(H_x, H_y, H_z)$ is vector sum of magnetic field components.

4.4. Command «s» stops the capture output. If any char or sequence not supported will be sent, the capture output will be stopped, but command reference will be received from magnetometer system as the answer. Sending the command «s» cause no answer need received from magnetometer system;

4.5. Command «f» save the current capture mode, interval of capture and calibration constants to internal FLASH in order to set them again at next launch of magnetometer system;

4.6. Command «3x» switches on all sensors to keep active state for simultaneously capturing the output. This is the fastest way to capture the measurements produced up to 20 3-axial measurements per second. However, each active sensor produces magnetic field, which causes other sensors output;

4.7. Command «1x» switch sensors to be captured sequentially in order to avoid magnetic fields interaction. Each time only 1 sensor active, other 2 sensors powered off. Only 3 measurements per second provided in this mode.

4.8. Commands to entering calibrating constants

Because FG-3+ sensor usually provide not linear output, the calibrated curve should be approximated by high-order function (see examples in section 6). A combined tangent function found as good balance between precision, calculation speed and easy calibration:

$$H_x = A_x + B_x * \tan(C_x / T_x + D_x);$$

$$H_y = A_y + B_y * \tan(C_y / T_y + D_y);$$

$$H_z = A_z + B_z * \tan(C_z / T_z + D_z);$$

where T_x , T_y , T_z are captured periods in units of MCU ticks, A_{xyz} , B_{xyz} , C_{xyz} , D_{xyz} are calibrated constants.

Because each FG-3+ sensor has unique shape of output function $H(T)$, the calibrated constants have different values as default.

4.9. Commands to set deviation tangents

Deviation tangents are tangents of angles fluxgate sensors axes are deviated from ideal orthogonal directions. It is known how difficult to install fluxgate sensors ideally on 90° from each other in the practice: every time even some small deviation occurred.

Sensor axes deviations produce an error when module of magnetic strength vector H calculated from H_x , H_y and H_z components measured. We can eliminate this error using simple mathematic technique described in section 6.

Let z-axis be always true, then actual values of H_x , H_y and H_z calculated from measured ones H_x^* , H_y^* and H_z^* using following expressions:

$$H_x = \frac{-H_x^* \cdot \sqrt{1 + t_{yx}^2 + t_{zx}^2} + H_y^* \cdot t_{yx} \cdot \sqrt{1 + t_{xy}^2 + t_{zy}^2} + H_z^* \cdot (t_{zx} - t_{zy} \cdot t_{yx})}{t_{xy} \cdot t_{yx} - 1}$$

$$H_y = \frac{H_x^* \cdot t_{xy} \cdot \sqrt{1 + t_{yx}^2 + t_{zx}^2} - H_y^* \cdot \sqrt{1 + t_{xy}^2 + t_{zy}^2} + H_z^* \cdot (t_{zy} - t_{zx} \cdot t_{xy})}{t_{xy} \cdot t_{yx} - 1} ;$$

$$H_z = H_z^*$$

where:

t_{xy} and t_{zy} – are tangents of angles between H_y^* vector projections to xy and yz planes;
 t_{yx} and t_{zx} – are tangents of angles between H_x^* vector projections to xy and xz planes.

4.10. Commands to calibrate the temperature sensor

To measure the temperature the NTC thermistor placed on the PCB. Because of NTC sensor usually provide not linear resistance output, the temperature should be calculated from simplified Steinhart – Hart equation:

$$R_{NTC}/R_{25} = \exp \cdot B(1/(t+273) - 1/298);$$

where R_{NTC} is resistance of NTC sensor measured by MCU at given temperature $t^\circ\text{C}$ ($t+273\text{K}$), R_{25} is resistance at 25°C (298K) taken from datasheet, B is Steinhart – Hart constant, also taken from datasheet for specified NTC sensor.

4.11. Command «int» sets the measurement interval for simultaneous mode in milliseconds.

4.12. Command «def» sets the calibration constants and deviation tangents to default values fabrication programmed.

4.13. Command «p» prints out current values of calibration constants and deviation tangents.

5. Recording the magnetic data (extended design only)

Extended design of simple 3-axis fluxgate magnetometer system provides the possibility of recording magnetic field strength components measured into uSD card with GPS coordinates binding and afterwards to transfer data captured to PC for further analysis using e.g. Surfer Software.

A lot of records could be produced continuously while free place in uSD card still available. Each new one starting immediately when record switch (fig. 2-2) was switched ON, and continuing until it will not be switched OFF.

Each record is a text file has name «DDMMYY_hhmmss.csv», where DD/MM/YY and hh:mm:ss are date and time of the moment record started. Record file has the following basic structure useful to be passed into Surfer Software:

	A	B	C	D	E	F	G	H
1	UTC time	Latitude,N	Longitude,E	Height,M	Hx,nT	Hy,nT	H _z ,nT	t,°C
2	19:12:36	5559.399414	5442.665527	213.699997	-9568,4	-8336,9	32229,4	15,6
3	19:12:36	5559.399414	5442.665527	213.699997	-9553,6	-8297,2	32229,4	15,6
4	19:12:36	5559.399414	5442.665527	213.699997	-9560,4	-8298,1	32253,7	15,6
5	19:12:36	5559.399414	5442.665527	213.699997	-9566,8	-8295,7	32253,7	15,5
6	19:12:36	5559.399414	5442.665527	213.699997	-9576,3	-8307,6	32240,0	15,6
7	19:12:36	5559.399414	5442.665527	213.699997	-9567,4	-8303,6	32240,0	15,6
8	19:12:36	5559.399414	5442.665527	213.699997	-9567,3	-8296,2	32302,6	15,6
9	19:12:36	5559.399414	5442.665527	213.699997	-9577,7	-8296,2	32302,6	15,6
10	19:12:36	5559.399414	5442.665527	213.699997	-9579,6	-8322,0	32275,0	15,6
11	19:12:36	5559.399414	5442.665527	213.699997	-9585,4	-8334,3	32275,0	15,6
12	19:12:36	5559.399414	5442.665527	213.699997	-9585,3	-8325,9	32275,0	15,6
13	19:12:36	5559.399414	5442.665527	213.699997	-9579,2	-8342,6	32250,5	15,6
14	19:12:37	5559.398438	5442.666016	214.600006	-9577,6	-8335,6	32250,5	15,6
15	19:12:37	5559.398438	5442.666016	214.600006	-9579,4	-8315,3	32260,2	15,6
16	19:12:37	5559.398438	5442.666016	214.600006	-9578,3	-8322,4	32260,2	15,6
17	19:12:37	5559.398438	5442.666016	214.600006	-9569,1	-8324,1	32266,4	15,6
18	19:12:37	5559.398438	5442.666016	214.600006	-9577,4	-8314,3	32266,4	15,6
19	19:12:37	5559.398438	5442.666016	214.600006	-9572,6	-8324,9	32250,0	15,6
20	19:12:37	5559.398438	5442.666016	214.600006	-9576,6	-8332,5	32250,0	15,6
21	19:12:37	5559.398438	5442.666016	214.600006	-9573,1	-8311,1	32261,4	15,6
22	19:12:37	5559.398438	5442.666016	214.600006	-9575,7	-8315,7	32261,4	15,7
23	19:12:37	5559.398438	5442.666016	214.600006	-9576,3	-8315,7	32236,1	15,6
24	19:12:37	5559.398438	5442.666016	214.600006	-9576,0	-8326,7	32236,1	15,6
25	19:12:37	5559.398438	5442.666016	214.600006	-9576,3	-8314,8	32215,0	15,6
26	19:12:37	5559.398438	5442.666016	214.600006	-9569,4	-8310,2	32215,0	15,6

Fig.5-1. Structure of magnetic field data record file

A green and blue LEDs together tell us is magnetometer ready for measurements.

Green (Yellow) PWR LED* indicating if fault of the power state occurred and record state. It has 3 states: quiescent, flashing, and active.

Quiescent indicates that power supply is good.

Flash indicates the command was received now from USART or/and record to uSD card is active.

Active indicates supply voltage out of range, may be occurred if supply accumulator was discharged, or supply voltage higher than 16V was applied to system. When supply voltage is in the right range, the system starts working normally again and a new record is automatically created to save new magnetic data, if the record switch is not switched OFF. During this the LED will switch to quiescent state.

If fault of the power state occurs during active recording of the magnetic data, the recording stops and the current record file closes until supply voltage returns into the right range.

Blue GPS LED indicating if the data from GPS receiver are valid. It has 4 states:

- long triple flashing every 4 seconds indicating that GPS data are not valid yet because of not enough satellites detected yet;

- short single flashing blue LED every 1 second indicating that GPS data are valid and ready to be recorded together with magnetic data;

- active LED (continuously on while record switch is active) indicates the data recording fault. Data recording should be switched OFF and uSD card need to be checked;

Quiescent state of both LEDs together indicates the system is not working, since no power supply applied or uSD card was not initialized at the system launch.

To record the magnetic data follow the sequence of steps below

5.1. Power ON the magnetometer system, check if green LED in quiescent state, wait until blue LED will produce short single flashes every 1 second;

5.2. Switch ON the record switch to start recording;

5.3. Switch OFF the record switch to stop recording;

5.4. Connect magnetometer system to PC using USB A(M) – A(M) cable, wait until PC operating system automatically install required USB flash disk driver;

5.5. Open installed flash disk using file explorer, copy the records to PC;

5.6. Disconnect the magnetometer system from PC, power it OFF.

* = old FG-33+ have green newer have yellow LED

6. Solving sensor axes deviation problem

Main advantage of 3-axis fluxgate magnetometer system is simultaneous measuring both magnetic field strength vector components H_x , H_y , H_z , and absolute value of magnetic field vector: $H = \sqrt{H_x^2 + H_y^2 + H_z^2}$.

To measure magnetic field strength vector components H_x , H_y , H_z directly we have to place fluxgate sensors ideally orthogonally to each other. In the practice magnetic field sensor axis has some deviation from ideal orientation, so we need to get exact expressions to calculate true magnetic field strength vector components H_x , H_y , H_z from measured values.

Now let's calculate the true H_z value from H_z^* actually measured for the z-axis only, then distribute the result obtained to other axes. Let's presume the angles between H_z^* vector projections to xz and yz planes are known, let's call them α_{xz} and α_{yz} respectively (fig.6-1).

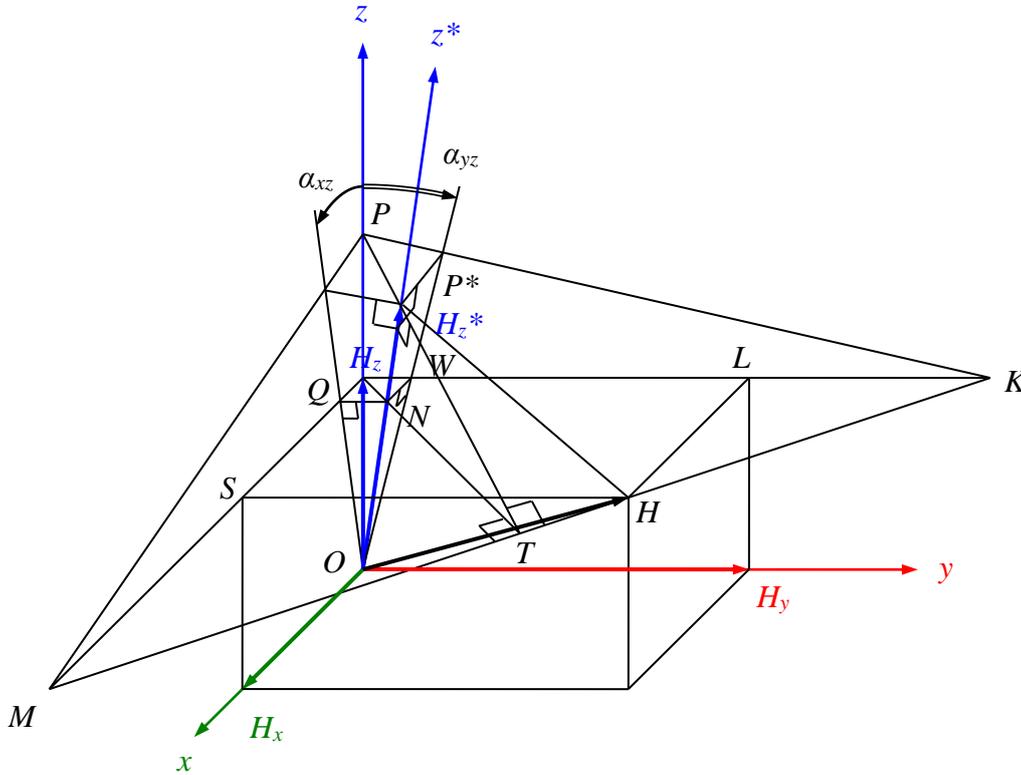


Fig.6-1. When axis of fluxgate sensor z^* deviates from true z -axis

So, we have to find relation between H_z and H_z^* depending on γ and β parameters.

$$H_z^* = OP^* = ON + NP^* \quad (6.1)$$

$$ON = \sqrt{OH_z^2 + NH_z^2} = \sqrt{OH_z^2 + QH_z^2 + WH_z^2} = H_z \sqrt{1 + tg^2 \alpha_{xz} + tg^2 \alpha_{yz}} \quad (6.2)$$

Let's, ON is calculated now. The calculation of NP^* will require a bit more effort:

$$\frac{NP^*}{NT} = \frac{NH_z}{ON} \Rightarrow NP^* = \frac{NT \times NH_z}{ON} \quad (6.3)$$

$$\begin{aligned} H_z Q &= H_z \operatorname{tg} \alpha_{xz} & H_z W &= H_z \operatorname{tg} \alpha_{yz} \\ H_z S &= H_x & H_z L &= H_y \\ SM &= H_z \frac{\operatorname{tg} \alpha_{yz}}{\operatorname{tg} \alpha_{xz}} & LK &= H_z \frac{\operatorname{tg} \alpha_{xz}}{\operatorname{tg} \alpha_{yz}} \end{aligned} \quad (6.4)$$

$$NH_z = \sqrt{QH_z^2 + WH_z^2} = H_z \sqrt{\operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}} \quad (6.5)$$

Let's, ON and NH_z are calculated now. Only NT calculation required in order to determine NP^* from expression 6.3 in terms of α_{xz} , α_{yz} , and H_z .

$$NT = TH_z - NH_z = MH_z \cos(MH_z T) - NH_z = MH_z \cos(NH_z Q) - NH_z \quad (6.6)$$

$$\begin{aligned} MH_z &= H_x + H_y \frac{\operatorname{tg} \alpha_{yz}}{\operatorname{tg} \alpha_{xz}} \\ NH_z &= H_z \sqrt{\operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}} \\ \cos(MH_z T) &= \frac{QH_z}{NH_z} = \frac{\operatorname{tg} \alpha_{xz}}{\sqrt{\operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}}} \end{aligned} \quad (6.7)$$

Insert expressions (6.7) into (6.6) to obtain NT :

$$NT = \frac{H_x \operatorname{tg} \alpha_{xz} + H_y \operatorname{tg} \alpha_{yz}}{\sqrt{\operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}}} - H_z \sqrt{\operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}} \quad (6.8)$$

Insert expressions (6.2), (6.5), and (6.8) into (6.3) to obtain NP^* :

$$NP^* = \frac{H_x \operatorname{tg} \alpha_{xz} + H_y \operatorname{tg} \alpha_{yz} - H_z (\operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz})}{\sqrt{1 + \operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}}} \quad (6.9)$$

Insert expressions (6.2) and (6.9) into (6.1) to obtain desired H_z^* :

$$H_z^* = H_x \frac{\operatorname{tg} \alpha_{xz}}{\sqrt{1 + \operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}}} + H_y \frac{\operatorname{tg} \alpha_{yz}}{\sqrt{1 + \operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}}} + H_z \frac{1}{\sqrt{1 + \operatorname{tg}^2 \alpha_{xz} + \operatorname{tg}^2 \alpha_{yz}}} \quad (6.10)$$

Check some special cases:

If $\alpha_{xz} = \alpha_{yz} = 0$ then $H_z^* = H_z$;

If $\alpha_{xz} = 0$ then $H_z^* = H_y \frac{\operatorname{tg} \alpha_{yz}}{\sqrt{1 + \operatorname{tg}^2 \alpha_{yz}}} + H_z \frac{1}{\sqrt{1 + \operatorname{tg}^2 \alpha_{yz}}}$;

If $\alpha_{yz} = 0$ then $H_z^* = H_x \frac{\operatorname{tg} \alpha_{xz}}{\sqrt{1 + \operatorname{tg}^2 \alpha_{xz}}} + H_z \frac{1}{\sqrt{1 + \operatorname{tg}^2 \alpha_{xz}}}$.

Now look at the most general case when all 3 sensor axes deviate from the true x -, y -, and z -axis. Let's presume angles between H_x^* vector projections to xz and xy planes are α_{zx} and α_{yx} respectively, angles between H_y^* vector projections to yz and xy planes are α_{zy} and α_{xy}

Since axis of one of fluxgate sensor can be used as true, let sensor z^* to coincide with true z -axis, so $\alpha_{xz} = \alpha_{yz} = 0$, $H_z^* = H_z$. From expression (6.10) we obtain $H_{xyz}^* \rightarrow H_{xyz}$ transform:

$$\begin{aligned} H_x^* &= H_x f(\alpha_{yx}, \alpha_{zx}) + H_y g(\alpha_{yx}, \alpha_{zx}) + H_z f(\alpha_{zx}, \alpha_{yx}) \\ H_y^* &= H_x f(\alpha_{xy}, \alpha_{zy}) + H_y g(\alpha_{zy}, \alpha_{xy}) + H_z f(\alpha_{zy}, \alpha_{xy}) \\ H_z^* &= H_z \end{aligned}$$

where

$$\begin{aligned} f(\alpha_1, \alpha_2) &= \frac{tg \alpha_1}{\sqrt{tg^2 \alpha_1 + tg^2 \alpha_2 + 1}} \\ g(\alpha_1, \alpha_2) &= \frac{1}{\sqrt{tg^2 \alpha_1 + tg^2 \alpha_2 + 1}} \end{aligned}$$

And, skipping intermediate calculations, inverse transform $H_{xyz} \rightarrow H_{xyz}^*$ expressions:

$$\begin{aligned} H_x &= \frac{-H_x^* \sqrt{1 + tg^2 \alpha_{yx} + tg^2 \alpha_{zx}} + H_y^* \sqrt{1 + tg^2 \alpha_{xy} + tg^2 \alpha_{zy}} + H_z^* (tg \alpha_{zx} - tg \alpha_{zy} tg \alpha_{yx})}{1 - tg \alpha_{xy} tg \alpha_{yx}} \\ H_y &= \frac{H_x^* \sqrt{1 + tg^2 \alpha_{yx} + tg^2 \alpha_{zx}} - H_y^* \sqrt{1 + tg^2 \alpha_{xy} + tg^2 \alpha_{zy}} + H_z^* (tg \alpha_{zy} - tg \alpha_{zx} tg \alpha_{xy})}{1 - tg \alpha_{xy} tg \alpha_{yx}} \\ H_z &= H_z^* \end{aligned} \quad (6.12)$$

Deviation tangents $tg \alpha_{zx}$, $tg \alpha_{yx}$, $tg \alpha_{zy}$, $tg \alpha_{xy}$, can be measured directly or fitted using differential evolution technique.